LUNCH BUNCH
September 24, 1997

The Black-Scholes Option Pricing Formula

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1. The Hedging Portfolio in Discrete Time

Simplest Example

HAL stock is currently at \$100. One month from now it will either be at \$110 (with probability p) or at \$90 (with probability $q=1-p$). How much is the option to buy HAL stock at \$95 one month from now worth today? Assume the risk free rate of interest is 0%. Call the option C, for call. C is a European option because it does not allow for early exercise. Assume through out that all securities are infinitely divisible and that stocks do not pay dividends.

Solution

One approach to pricing C is to set up a portfolio of stocks and cash to replicate C's payout. Such a portfolio is called a **Hedging Portfolio**. The hedging portfolio can be long or short in both cash and the stock. Long means we own stock or cash, short means we owe stock or cash. The cost of setting up the hedging portfolio is then a reasonable price for C. To form a hedging portfolio we thus must hold a stocks and b cash so that:

$a \times 110 + b = 15$	C is worth \$15 if the stock moves to \$110
$a \times 90 + b = 0$	C is worth nothing if the stock moves to \$90.

Thus $a = 15/20 = 3/4$ and $b = -\$67.50$. In order to set this portfolio up today, we need to buy 3/4 of one stock and take out a loan for \$67.50; b being negative corresponds short cash, i.e. to debt. If we are given \$7.50 we can borrow \$67.50 giving \$75 which is exactly enough to buy 3/4 of a stock. Since the resulting portfolio exactly replicates the payout of the option, with no risk, C must be worth \$7.50 today.

If C sold for more than \$7.50, say for \$7.75, we would write (sell) options. We would get \$7.75 per option, and we would use \$7.50 to set up a hedging portfolio. The result would be a risk-free arbitrage profit of \$0.25 per option.

Conversely, if options sold for \$7.25 we could short a stock to get \$100. We would use the \$100 to buy a 4/3 options and put \$90 1/3 in the bank. If stock prices rise to \$110 when the options expire, we exercise them to yield a \$20 profit. Combined with the \$90 1/3 in the bank this gives \$110 1/3, which is enough to purchase one stock to close out the short position with \$1/3 left over as a risk free profit. If stock prices fall to \$90 the option expires worthless but he have \$90 1/3 in the bank to close the short position and still make a risk free profit of \$1/3. This is the essence of **arbitrage-free pricing**.

Subtle Exercise 1

What is the fallacy in the above argument?

Next Simplest Example

As above, only assume the risk free interest rate $r = 10\%$ compounded continuously. Recall from 4A that the interest rate in a period t (measured in years) is $\exp(0.10t)$ and the discount factor is $\exp(-0.10t)$.

Solution

Again, we want to set up the hedging portfolio. The two equations for a and b are the same. The only difference is that to owe \$67.50 one month from now would only cost $\$67.50 \times \exp(-0.10/12) = 66.94$ today because the bank would be good enough to charge us interest. Thus we would need an extra \$8.06 cash now to set up the hedging portfolio. \$8.06 is therefore a fair price for the option today. Charging interest has made the option more expensive because the hedging portfolio involves assuming debt.

Notice that in both examples the amount of stock in the hedging portfolio, 3/4, is given by (change in price of option) / (change in price of stock). Below we will see that the continuous-time hedging portfolio involves holding an amount of stock equal to the partial derivative of the option price with respect to the call. The partial derivative is the limit of (change in price of option) / (change in price of stock) over very short time periods.

What is missing?

The solution to the option pricing problem does not depend on p and q! In fact, we can use the call option price to back-into probabilities p and q by arguing that the call value is the expected value of the payout.

When $r = 0\%$ we get $C = 7.50$ = Expected Payout of Option $= p \times 15 + q \times 0$, so $p = 0.5$ and $q = 0.5$. The return on the stock now also equals the risk free return of 0% because the expected end of period price is \$100.

When $r = 10\%$ the expected value of the call at maturity is $\exp(0.10/12) \times 8.06 = 8.13 = p \times 15 + q \times 0$, so $p = 0.542$ and $q = 0.458$. The expected stock price one month from now $(110 \times 0.542 + 90 \times 0.458) = 100.837$, giving a one period return of 0.833% or an annual return of 10%, which also equals the risk free rate of return. Coincidence? I don't think so.

Why is the implied rate of return the risk free rate?

Because the option price is independent of investor risk profiles! We have figured out a risk-free price using the hedging portfolio. The price is therefore independent of risk preferences and so we can assume that all investors are risk-neutral. Risk-neutrality is one particular risk preference. In a risk-neutral world all stocks are *expected* to earn the risk free rate of return because there is no need to pay investors a risk premium.

Other types of Options

So far we have mentioned a **European call** option which is the right to buy a stock for a fixed price at a certain date in the future. The payoff from a European call at expiration is $\max(S-E,0)$ where S is the ending stock price and E is the exercise price. This is the same as the formula for losses excess of E, where S is the loss random variable.

A **European put** is like a European call except that it gives you the right to sell stock at the exercise price. (I always remember this as **B**uy—**C**all, **P**ut—**S**ell is in alphabetical order.) A put is worth $\max(E-S,0)$ at expiration. In insurance, the put option computes the value of a profit commission.

In contrast to European options, **American options** allow for early exercise. The Black-Scholes formula, and everything in these notes, applies to European options. It may, or may not, apply to American options.

2. Properties of the Stock Price Process

In order to develop Section 1 into a more sophisticated model we need to know more about the stock price process. Section 2 develops both the necessary theory and gives an example using the S&P 500 index.

2.1 Theory

In Section 1 we assumed a 1 month time frame and additive increments in stock price. We now want to generalize that simple binomial model by letting the time steps get smaller and smaller. We want to do this in such a way that the expected return over a short period of time dt is approximately normal with mean μdt and standard deviation $\sigma\sqrt{dt}$.

Suppose the stock price is currently S and that in a short time period dt the price can move up to Su ($u > 1$) with probability p, or down to Sd, $d = 1/u$ with probability $q = 1-p$. Assume $p > q$, so the stock has a positive expected return. This model is called a recombining binomial tree. (The change from the additive of Section 1 to multiplicative model is just a convenience.) If we set

$$u = \exp(\sigma\sqrt{dt}), \quad p = \frac{\exp(\mu\sqrt{dt}) - d}{u - d}$$

the our requirements for the mean and variance are met. The equation for p is similar to the one implicit in the calculations in Section 1. These assumptions imply the upwards increment in stock price is approximately

$$Su = S\exp(\sigma\sqrt{dt}) \approx S(1 + \sigma\sqrt{dt} + \sigma^2 \frac{dt}{2})$$

ignoring terms higher than dt, and similarly for downward movements. Thus in time dt the stock price moves about $\sqrt{dt}$ which, as dt is small, is a *much* larger quantity. Below we will use the following key fact which we can derive from the above representation of Su:

$$(S - Su)^2 = (\Delta S)^2 \approx (\sigma\sqrt{dt})^2 = \sigma^2 dt .$$

2.2 Practice

The next three figures show the recent history of the S&P 500 index.

FIGURE 1: S&P 500 INDEX CLOSING PRICE, EXPONENTIAL FIT AND 75 DAY MOVING AVERAGE STANDARD DEVIATION

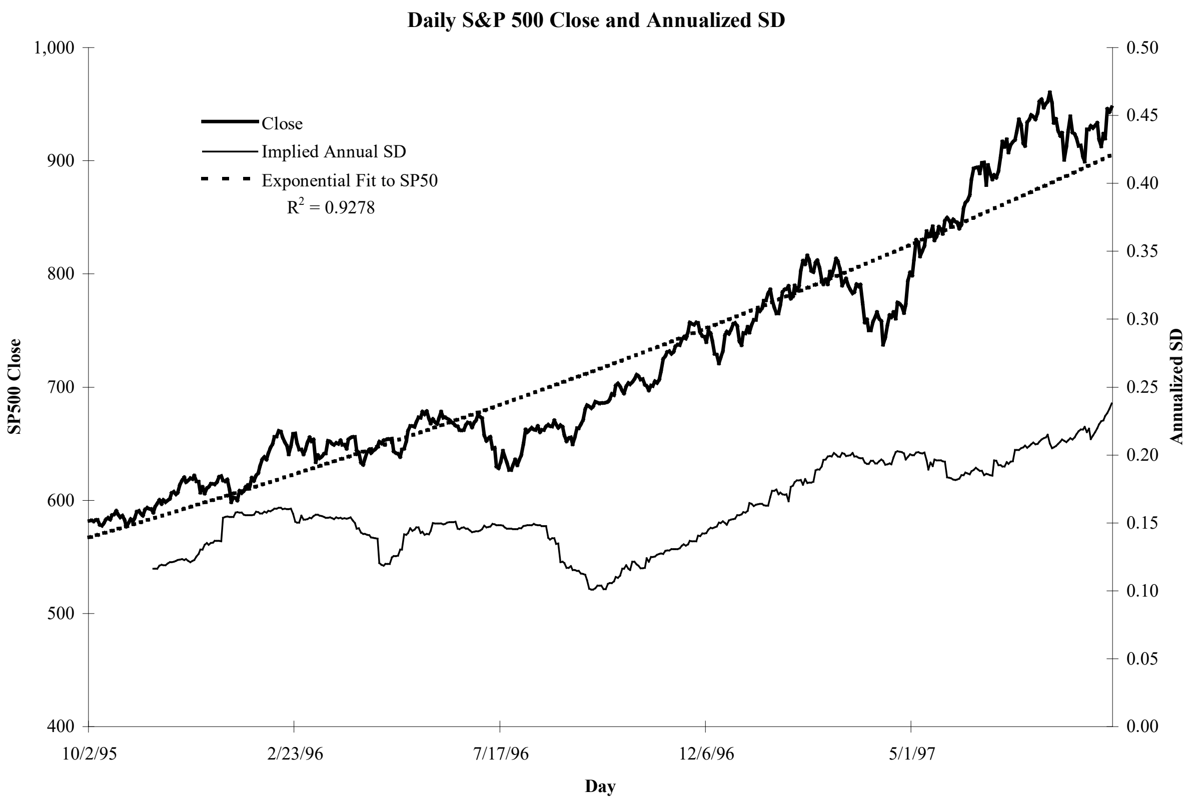


FIGURE 2: S&P 500 DAILY RETURNS AND MOVING AVERAGE STANDARD DEVIATION

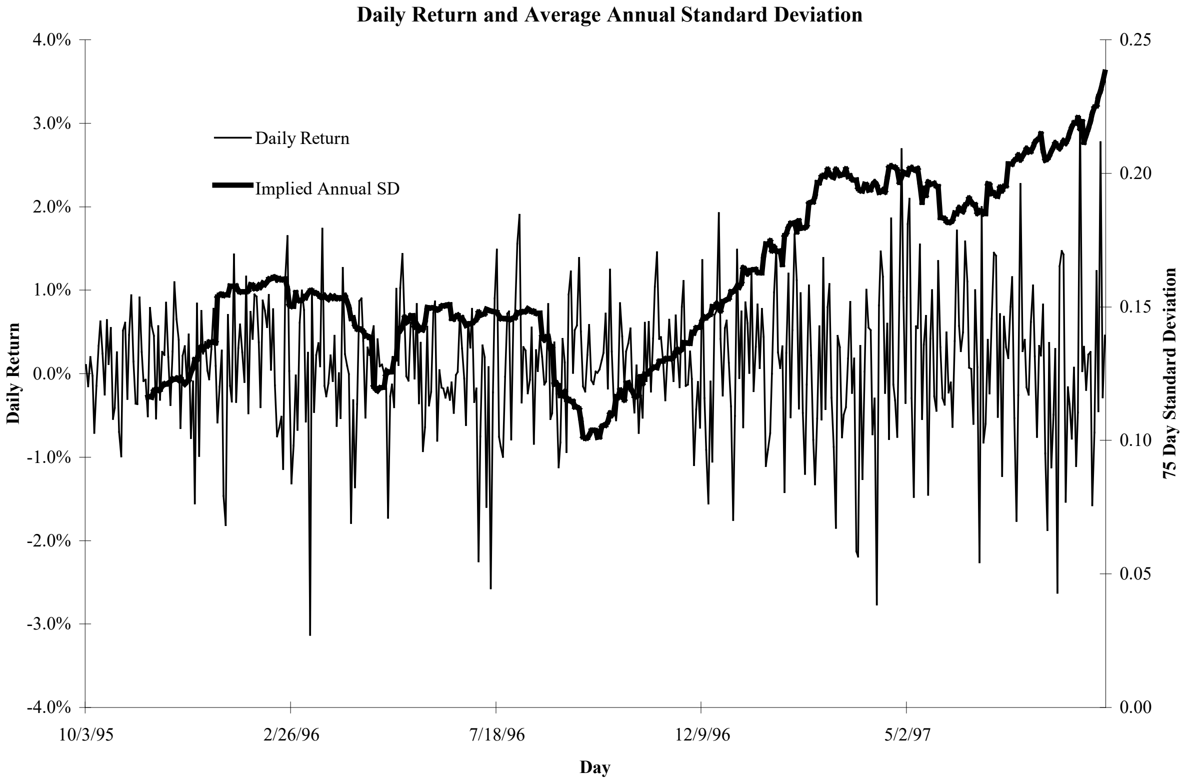
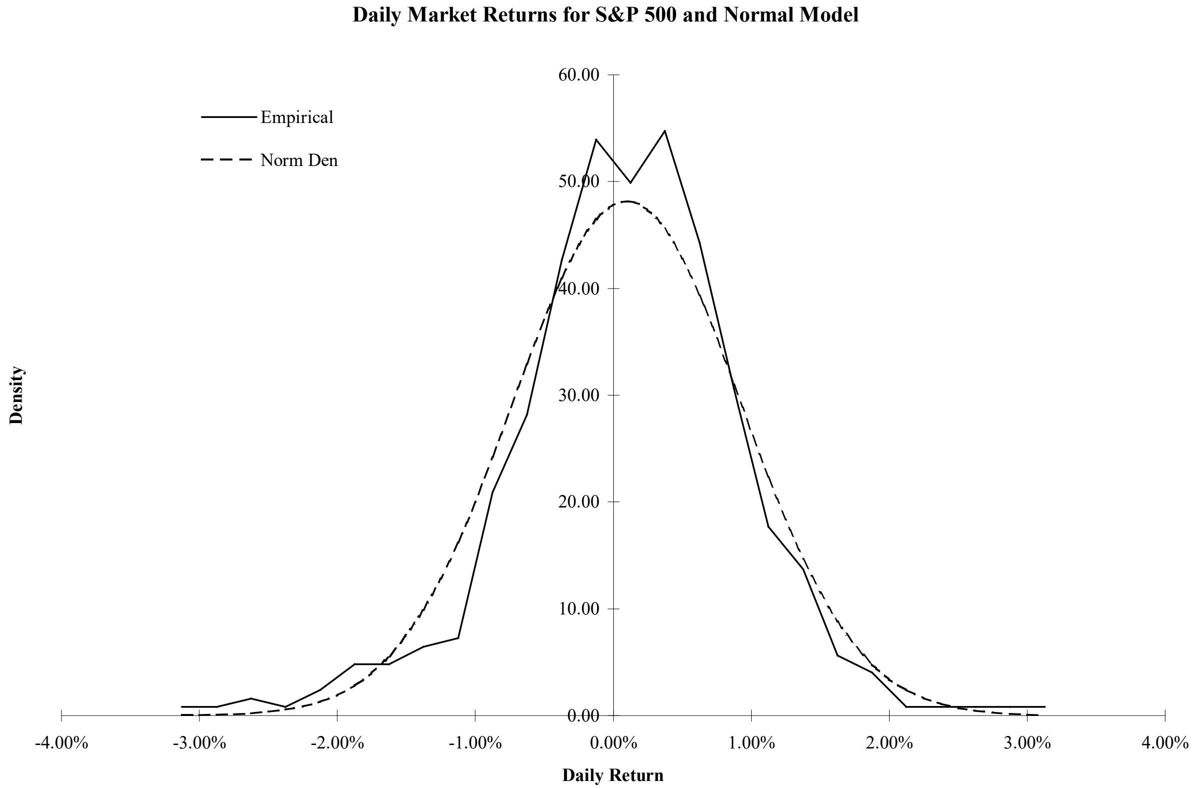


FIGURE 3: NORMAL FIT TO S&P 500 DAILY RETURN



The normal fit is not great, but not terrible, with a chi-squared p value of 3.8%. Notice this particular series of S&P prices is actually negatively skewed, which is quite unusual.

Given these observations, we will model the stock price using a **geometric Brownian motion**. We assume the stock price process satisfies

$$dS_t = \mu S_t dt + \sigma S_t dW_t .$$

Here dW is a normal noise term with mean zero and variance dt. The average log return over a short period of time period is μdt and standard deviation of returns is $\sigma\sqrt{dt}$. In terms of prices, this gives

$$S_t = \exp((\mu - \sigma^2 / 2)t + \sigma W_t)$$

where $W(t)$ is normally distributed with mean zero and variance t. It is clear that $S(t)$ is lognormal with expected value $\exp(\mu t)$.

Exercise

Try to derive the above formual for $S(t)$.

3. The Hedging Portfolio in Continuous Time and the Black-Scholes Partial Differential Equation

We now construct a hedging portfolio in continuous time. The idea is the same as in Section 1: look for some combination of stock and cash (actually debt) that duplicates the payout of the option. The cost of initially setting up this portfolio will be the option price, by arbitrage. Assume the stock price at time t from expiration is initially S_t and let *r* be the risk free rate of return.

We will write the call price as *C*(*S*,*t*) since it is a function of the current stock price *S*, and the time to expiration *t*. Now, using a Taylor's series argument, we get

$$\begin{aligned} C(S,t) &= C(S_0,t_0) + \frac{\partial C}{\partial S}(S-S_0) + \frac{\partial C}{\partial t}(t-t_0) + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(S-S_0)^2 \\ &= C + \frac{\partial C}{\partial S}\Delta S + \frac{\partial C}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\Delta S^2. \end{aligned}$$

Since we know $dS_t = \mu S_t dt + \sigma S_t dW_t$ we can use the key fact noted at the end of Section 2.1 to get

$$\begin{aligned} \Delta C &= \frac{\partial C}{\partial S}(\mu S_t \Delta t + \sigma S_t \Delta W) + \frac{\partial C}{\partial t}\Delta t + \frac{1}{2}S_t^2\frac{\partial^2 C}{\partial S^2}(\mu^2(\Delta t)^2 + 2\mu\sigma(\Delta t)(\Delta W) + \sigma^2(\Delta W)^2) \\ &= (\mu S_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C}{\partial S^2})\Delta t + \sigma S_t \frac{\partial C}{\partial S}\Delta W. \end{aligned}$$

The important thing about this equation is that the stochastic term, $\sigma S_t \frac{\partial C}{\partial S}\Delta W$, is $\frac{\partial C}{\partial S}$ times the stochastic term $\sigma S_t \Delta W_t$ of the stock price. This means that if we have a portfolio with $\frac{\partial C}{\partial S}$ stocks on one side and one option on the other then the stochastic terms will cancel out. In analogy with Section 1 this is the hedging portfolio.

The hedging portfolio is therefore $\Pi = C - \frac{\partial C}{\partial S}S$. Since it is risk free (has no stochastic term), it must earn the risk free rate of return: $\Delta\Pi = r\Pi\Delta t$. Computing, we get

$$\begin{aligned} \Delta\Pi &= \Delta C - \frac{\partial C}{\partial S}\Delta S \\ &= (\mu S \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2})\Delta t + \sigma S \frac{\partial C}{\partial S}\Delta W - \frac{\partial C}{\partial S}(\mu S \Delta t + \sigma S \Delta W) \\ &= \left(\frac{\partial C}{\partial t} + \frac{\sigma^2}{2}S^2\frac{\partial^2 C}{\partial S^2}\right)\Delta t = r\Pi\Delta t \\ &\Rightarrow \frac{\partial C}{\partial t} + \frac{\sigma^2}{2}S^2\frac{\partial^2 C}{\partial S^2} = r(C - S\frac{\partial C}{\partial S}) \\ &\Rightarrow rS\frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{\sigma^2}{2}S^2\frac{\partial^2 C}{\partial S^2} - rC = 0 \end{aligned}$$

which is the famous Black-Scholes partial differential equation. The BS PDE is **independent** of μ. Just as in the finite case, we have an equation for the option price which does not depend on risk-preferences. Again, this should be plausible because there is no risk to the investor of writing the option because it can all be hedged away. In insurance it is generally not possible to hedge losses in this way, so the equation is of dubious applicability.

We could now solve the BS PDE using numerical methods and derive a solution. We know the ending condition as $C(S,0)=\max(S-E,0)$. However, in the next section we will exhibit an explicit solution.

The BS PDE was actually famous long before Black-Scholes from physics and quantum mechanics. It was originally solved by Feynman and Kac.

4. Explicit Solution to the Black-Scholes PDE

Let's step back from this and try to figure out how much an actuary would charge for the call option C. In essence we have the following insurance contract:

- Loss distribution is lognormal with (lognormal) parameters μt and σt, where t is the time to expiration of the option
- Risk free rate of return r
- Policy has an AAD equal to E, the exercise price ($100 in the Section 1 examples).
- Losses will be known with certainty time t from now and will be paid immediately. Therefore there is no issue with loss development and the loss discount factor is $\exp(-rt)$.

As a first guess an actuary would price this as

$$\exp(-rt)E(\max(0, X - E))$$

where X is a lognormal random variable with parameters μt and σt. The actuary would then have to select μ and σ. The parameters could be estimated from a time series of daily stock price closes (as in Section 3) and / or derived from some financial model such as the CAPM.

As a first approximation, the actuary might try ignoring the risk premium in the stock and assume $\mu = r$. Then he or she would get the following expression for the call option:

$$C(S,t,E,r,\sigma) = SN(d_1) - Ee^{-rt}N(d_2), \text{ where}$$

$$d_1 = \frac{\ln(S / E) + (r + \sigma^2 / 2)t}{\sigma\sqrt{t}}, \quad d_2 = d_1 - \sigma\sqrt{t}.$$

While you may not carry this formula around in your head you could compute it (or look it up in the Appendix to Hogg and Klugman, p. 229).

Miracle:

The risk-free solution to the simple binomial model and the fact that the BS PDE does not depend on the return parameter μ should prepare us for the fact that the above equation is a solution to the BS PDE and therefore gives the price of the call! Therefore there is no need to worry about determining μ.

5. Properties of the Black-Scholes formula

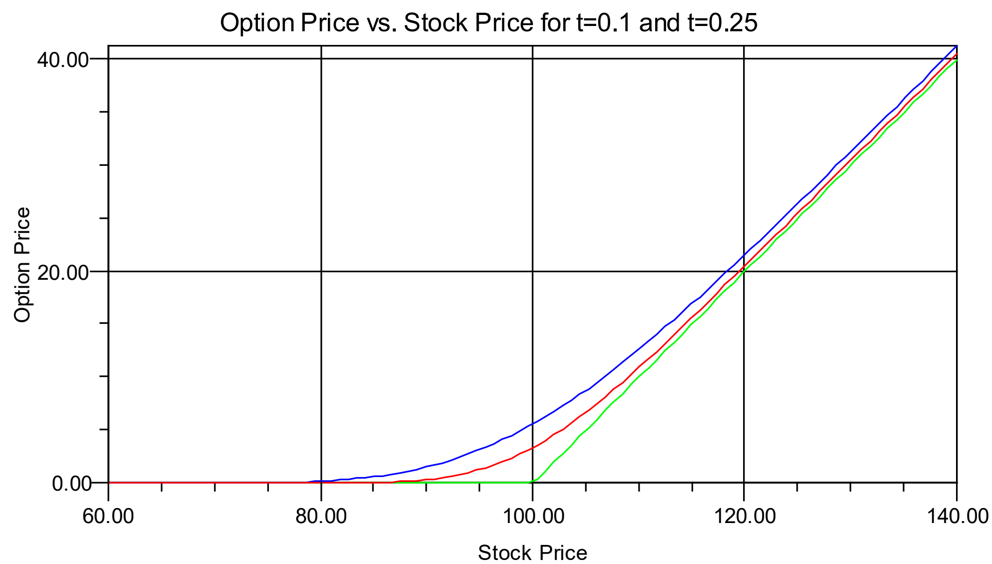
From the BS formula we see the price is determined by five variables: the current stock price S, the time to expiration t, the exercise price E, the risk free rate of interest r, and the volatility of the stock σ.

5.1 Impact of a change in individual pricing variables

In the following examples $E = 100$, $r = 0.05$ and other variables are as given.

Stock price

An increase in the stock price increases the value of the call option: it becomes more "in the money" and more likely to be exercised. Analogously, we charge more for losses excess of a given AAD the greater expected losses. The figure below shows the option value at expiration (angled green line) and at time 0.1 years (middle red line) and 0.25 years (top blue line) from expiration. Note that the price at expiration is max(S–E,0) as it should be.



Exercise Price

An increase in the exercise price decreases the value of the call; indeed the stock price and the strike price have complementary roles. In insurance, increasing the AAD decreases the premium.

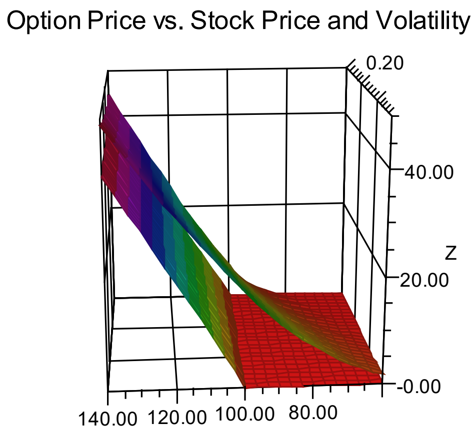
Risk Free Rate of Interest

An increase in the risk free rate of interest leads to an increase in the call price. We saw this in the first two examples of Section 1.

In insurance the opposite is true: as discount rates increase premiums decrease. However, in insurance we would observe there is a correlation between the discount rate and the rate of inflation on liabilities. Inflation on liabilities is analogous to the return on the stock. Thus, if we increase the discount rate should also increase the expected liabilities (μt term). If we do this then we recover the option result. (In terms of the BS formula, the way an actuary would increase the discount factor would only affect the exp($-rt$) term; in reality it would affect the two $N(d)$ terms as well.)

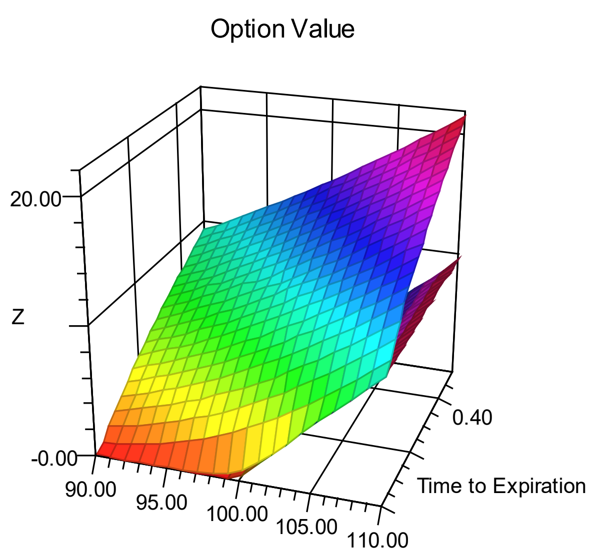
Volatility of Stock

An increase in the volatility of the stock price process increases the price of the option. This is analogous to the fact that a profit commission is worth more to the cedent on a volatile book of business than it is on a less volatile book. The figure below shows stock price on the horizontal axis, volatility decreasing front to back from 0.40 to 0.20 and the option price on the vertical axis.



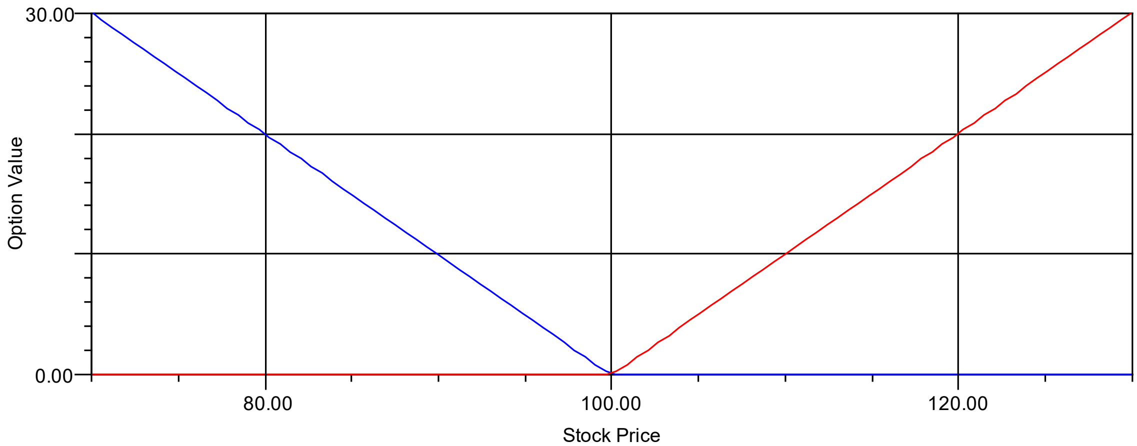
Time to Expiration

The figure below shows the effect of increasing time to expiration on the call option; clearly the value increases as the time to expiration increases—assuming that the stock does not pay dividends. If the stock does pay dividends then the European call does not necessarily increase in value with time to expiration.



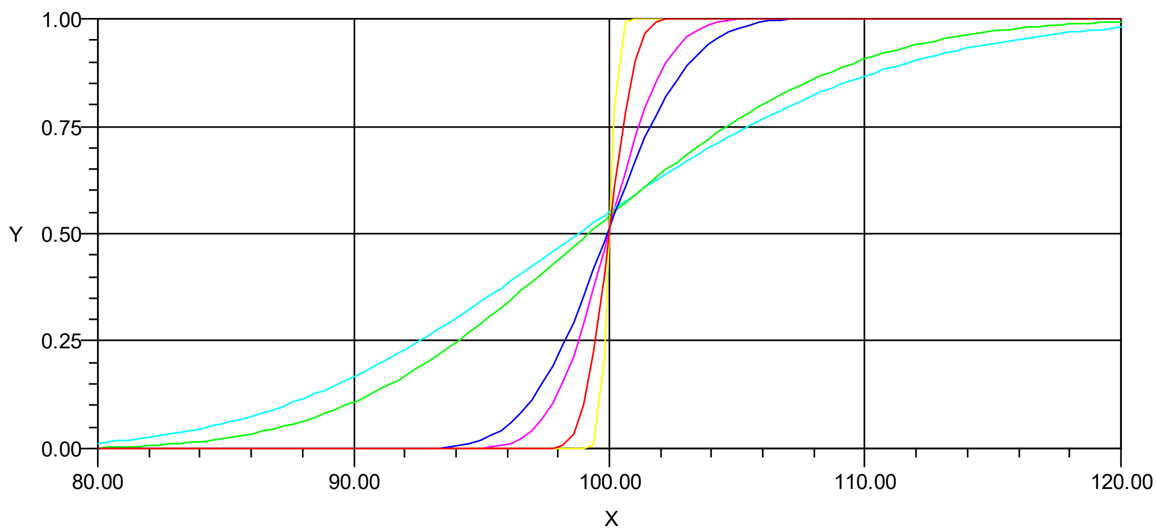
Put/Call Parity

Put call parity says that cash plus stock equals call minus put. In insurance it says that one plus the entry ratio equals the savings plus the expense. I leave the details to you, but here is the relevant profit diagram.



Graphical Representation of Hedge Portfolio

The next figure shows the derivative of the call price with respect to the stock price for various times to expiration. The *x*-axis shows the stock price and the *y*-axis shows the partial derivative. Notice that the derivative is always between zero and one (remember it gives the number of stocks to hold in the hedging portfolio for one call option) and that as the time to expiration gets smaller and smaller the curve jumps more and more quickly from near zero to near one. This corresponds to the option moving from out of the money (when the hedging portfolio does not need to hold a stock at expiration) to in the money (when the hedge portfolio does need a stock). Nearer expiration a faster adjustment to the hedging portfolio is required. The trading strategy implied by this is quite contrary: as stocks go up in price past the exercise price you buy, and when they go down you sell; the opposite of buy low and sell high! However, you only trade when the stock price is near the exercise price.



6. Market Pricing

So, how well does the Black-Scholes model perform? Exhibit 1 gives the market prices and BS prices for all puts and calls on the S&P 500 index per the Wall Street Journal for September 15, 1997. The first page gives the column headings etc. and the remaining pages give the prices for September, October, November puts and calls. Options expire on the third Friday of each month. Column (13) gives the percentage error in the option price and column

(15) gives the squared error. The exhibit uses a volatility of 23.5%. Bearing in mind there is only *one* free parameter here (since the risk free rate can be taken as known) the results are amazingly good. Comments are:

- Put prices are all derived using put / call parity.

- Some market prices are anomalies based on when the last transaction occurred. The S&P index may have moved substantially from when the last option traded to its final close. Examples are Sept. 890 calls.

- For way out of the money calls the market pricing does not go to zero as quickly as the model would indicate. This could be because: (1) trading is done in 1/16ths, (2) the market tails of the return distribution are thicker (the "smile").

- Column (14) shows the implied volatility. This is obtained by backing into σ based on the market pricing. Over most reasonable ranges of strike and exercise it is stable and in the range of recent empirical volatilities (see Section 2).

- There are options in the exhibit where the prices are given for 9/15 and9/16 (look at days to expiration).

7. Some Subtle Points

The derivation of the BS PDE sets up a portfolio and then computes the magnitude of a change in the portfolio. If you look carefully you will find an error in that step. It is not material.

The BS analysis assumes no-arbitrage. The following example, which is based on some well-known properties of Brownian motion, shows that unless we make some kind of assumptions about allowable trading strategies the Black-Scholes assumption of geometric Brownian motion for the stock price process actually *does* allow arbitrage.

Consider the following strategy to make \$1 over a unit time period. At $t = 0$, sell one stock short yielding $S(0)$, say. If the price of the stock goes down by \$1 or more buy back the stock and close the short position, yielding \$1. If by time $t = 1/2$ the stock price has not gone down by \$1 short more stock. If the price goes down by \$1 close out your whole position and (assuming you computed more correctly) you make \$1. If by time $t = 3/4$ the price is still up, short yet more stock and so forth. Because of some (well known) properties of Brownian motion, you are guaranteed to make your \$1 before time $t = 1$. However, you will need a very understanding broker because the strategy potentially involves taking arbitrarily large short positions, which are generally not allowed.

Answer to Subtle Question 1

In Section 1 we indicated there was an error in the derivation of the call price for the simplest example. The error is: what if we can construct a hedging portfolio for an amount other than \$7.50? In that case the arbitrage argument would not work.

If there is another hedging portfolio with a different initial cost then have two risk-free trading strategies which yield the same final payout. Their difference is a "suicide" strategy by which it is possible to guarantee a certain loss in a fixed period of time. The existence of such strategies messes up all arbitrage-free pricing because it implies prices are not unique. In the general BS formulation it is important to restrict trading strategies so that suicide strategies (which are basically the same as the arbitrage presented above) are not allowed. The solution to this problem involves some very sophisticated and subtle mathematics, but is probably of more interest to academics than practitioners.

What trading volume is needed for the Hedging Portfolio?

The suicide strategy and the short-sale arbitrage may seem contrived. However, the hedging portfolio implied by the BS model requires that an infinite amount of stock is traded in every time interval! This follows from the fact that the path of a Brownian motion has unbounded variation, that is, it travels an infinite distance (mostly small wiggles up and down) in any interval of time. This in turn follows from the key fact noted in Section 2.1, that in time dt it travels a distance proportional to $\sqrt{dt}$.

Addendum

During my talk, two objections were raised to the binomial option pricing examples given in Section 1. The objections were:

1. What if the up and down price movements were not symmetrical about the current price?

2. What if there is a 99% chance of an upward movement and 1% chance of a downward movement?

In this addendum I'll deal with these two objections. First, I want to write down the formulae for the important relationships. Notation:

- r is the one-period risk free interest rate
- S is the current stock price
- U is the stock price one period from now assuming an upwards movement
- D is the stock price one period from now assuming a downwards movement
- E is the exercise price of the call option
- C is the current value of the call option
- p is the probability of an up movement in the stock price and $q = 1 - p$
- p^* is the "risk-neutral" probability of an up movement in the stock price and $q^* = 1 - p^*$

In addition to these variables, which were all used in the talk, define:

- r_a is a risk adjusted interest rate, assumed strictly greater than r

In the talk, I dervied C by constructing a hedging portfolio and then showing by an arbitrage argument that C must equal the price of setting up the hedge. To do this we found the amount of stock a and amount of cash (bonds) b necessary to replicate the outcome of the option. If the stock moves to U then the call is worth $U^*:=\max(U - E, 0)$ and if the stock moves to D then the call is worth $D^*:=\max(D - E, 0)$. Thus a and b must satisfy the two equations

$$aU + b = U^*,$$
$$aD + b = D^*.$$

Solving these equations we get

$$a = \frac{U^* - D^*}{U - D}$$
$$b = U^* - aU = D^* - aD.$$

The value of the call is therefore the current cost of setting up a portfolio with a stocks and b bonds:

$$C = aS + \exp(-r)b.$$

This expression is clearly independent of p and also holds regardless of where S is between U and D.

I also showed in the talk that we could back into probabilities p^* and q^* relative to which S earned the risk free rate of return. These are called the risk adjusted probabilities. The key formula satisfied by p^* and q^* expressed C as the present value of the expected pay-off of the option:

$$C = \exp(-r)(p^*U^* + q^*D^*) = \exp(-r)(p^*(U^* - D^*) + D^*) .$$

Hence

$$p^* = \frac{\exp(r)C - D^*}{U^* - D^*} .$$

Now the expected price of the stock in one period is given by

$$\begin{aligned} p^*U + q^*D &= p^*(U - D) + D \\ &= (\exp(r)C - D^*)\left(\frac{U - D}{U^* - D^*}\right) + D \\ &= \exp(r)S + \left(\frac{U - D}{U^* - D^*}\right)(b - D^*) + D \\ &= \exp(r)S - \left(\frac{U - D}{U^* - D^*}\right)\left(\frac{U^* - D^*}{U - D}\right)D + D \\ &= \exp(r)S \end{aligned}$$

showing that the stock does earn the risk free return relative to p^* and q^*, as required.

The seemingly most compelling objection was, “what if the probability of an up movement equals 99%?” In the example in Section 1 we had $S = 100$, $U = 110$, $D = 90$, $E = 95$, and $r = 0\%$. The formula gives $C = \$7.50$. Naively, it seems C should be worth nearer $0.99 \times 15.00 = \$14.85$. As outlined in Section 1 the answer is that if the option sold for a price other than \$7.50 there would be arbitrage opportunities, and so, in the absence of arbitrage, C must still sell for \$7.50.

In order to address this strange result first consider how the current stock price is determined. According to the conventional wisdom S will be the present value of the expected future price of the stock discounted at a *risk adjusted* rate of interest:

$$S = \exp(-r_a)(pU + qD) .$$

We can solve this equation for r_a no matter what p, provided $p \neq 0,1$. In our case, where the stock is currently trading at 100, $r_a = 9.35\%$ if $p = 99\%$ indicating investors are quite risk averse. The objection to \$7.50 as the option price is that, surely, investors will see it as a good deal and hence buy. In fact, investors would not see it as a good deal because they are so risk averse. The option is always more leveraged than the stock, and so will appear riskier to investors. Also, they know about hedging and arbitrage.

It is interesting to see what happens if we assume (some) investors do not know about hedging. If they see the option as a good deal and buy it they will drive up the price of the stock. By buying options they force the seller (there would be queue at a price above \$7.50) to set up a hedging portfolio, which requires buying stock. Tempting though it is to try to use utility arguments and so forth to derive an equilibrium in the stock and option market, such an approach will be fruiteless because of the existence of the hedging portfolio.

The attached spreadsheet, lun_bseg.xls, gives a calculator to compute the the hedging portfolio and call option price given the inital, up and down stock prices and the risk free interest rate. It also computes the risk adjusted probabilities and the risk adjusted interest rate. As you can see by trying out different parameters, everything I've said is true.

Exhibit 1: Market Pricing vs. Black-Scholes Pricing

Unit of Time	365days
Data	WSJ, 9/16/97 and 9/17/97
Evaluation Date	9/15/97
3 month T-Bill	4.94%
Selected Disc Rate	5.25%
LIBOR, 3 month	5.72%
Force of Interest	5.12%
S&P 9/15 Close	919.77
S&P 9/16 Close	945.64

Option Expiration Dates	
Month	Exp Date
Dec	12/19/97
Nov	11/21/97
Oct	10/17/97
Sep	9/19/97

Col No.	**Heading**	**Explanation**
(1)	Exp Month	Expiration Month of Option
(2)	Exercise Price	Exercise Price of Option
(3)	Put/Call	p for put, c for call
(4)	Last Market Price	
(5)	Volume	Traded volume for 9/15/97 or 9/16/97
(6)	Exp Date	Expiration date, lookup from table, third Friday of month
(7)	Days to Exp	Number of days until option expires
(8)	Intrinsic Value	max(Index-Exercise,0) for calls, max(Exercise-Index,0) for puts
(9)	Time Value	Market price minus intrinsic value
(10)	Discount Factor	
(11)	BS Put Price (Put/Call Parity)	Put price derives from Put/Call parity and BS call price
(12)	BS Call Price	Black Scholes call formula price
(13)	BS Model Pricing Error	(BS price minus market price) / (market price)
(14)	Implied Volatility	Volatility implied by market price and BS formula
(15)	SSE	(Market price minus BS price)^2

Selected Volatility	23.50%
Call Min SSE	23.97%
Put Min SSE	22.88%

(1) Exp Month	(2) Exercise Price	(3) Put/ Call	(4) Last Market Price	(5) Volume	(6) Exp Date	(7) Days to Exp	(8) Intrinsic Value	(9) Time Value	(10) Discount Factor	(11) BS Put Price	(12) BS Call Price	(13) BS Model Pricing Error	(14) Implied Volatility	(15) SSE
Sep	700	c	220 1/2	372	9/19/97	4	219.77	0.73	0.99944	n/a	220.16	-0.2%	???	0.11
Sep	750	c	174 1/4	353	9/19/97	4	169.77	4.48	0.99944	n/a	170.19	-2.3%	???	16.48
Sep	790	c	133	3	9/19/97	4	129.77	3.23	0.99944	n/a	130.21	-2.1%	???	7.77
Sep	850	c	78 1/4	21	9/19/97	4	69.77	8.48	0.99944	n/a	70.25	-10.2%	???	64.00
Sep	870	c	56 1/4	1	9/19/97	4	49.77	6.48	0.99944	n/a	50.34	-10.5%	61.18%	34.90
Sep	875	c	50 1/2	2	9/19/97	4	44.77	5.73	0.99944	n/a	45.42	-10.1%	54.32%	25.77
Sep	880	c	40 1/2	15	9/19/97	4	39.77	0.73	0.99944	n/a	40.56	0.2%	22.54%	0.00
Sep	890	c	**39 3/4**	5	9/19/97	4	29.77	9.98	0.99944	n/a	31.16	-21.6%	56.33%	73.81
Sep	900	c	23 3/4	26	9/19/97	4	19.77	3.98	0.99944	n/a	22.50	-5.3%	28.14%	1.57
Sep	905	c	**25 1/4**	2	9/19/97	4	14.77	10.48	0.99944	n/a	18.59	-26.4%	43.31%	44.38
Sep	910	c	14 1/2	1,633	9/19/97	4	9.77	4.73	0.99944	n/a	15.04	3.7%	21.93%	0.29
Sep	915	c	13 1/2	1,182	9/19/97	4	4.77	8.73	0.99944	n/a	11.89	-11.9%	27.81%	2.60
Sep	920	c	8 3/4	1,353	9/19/97	4	0.00	8.75	0.99944	n/a	9.17	4.8%	22.41%	0.18
Sep	925	c	6 1/4	2,318	9/19/97	4	0.00	6.25	0.99944	n/a	6.89	10.2%	21.80%	0.41
Sep	930	c	4 1/4	5,866	9/19/97	4	0.00	4.25	0.99944	n/a	5.03	18.5%	21.25%	0.62
Sep	935	c	2 1/2	2,209	9/19/97	4	0.00	2.50	0.99944	n/a	3.57	43.0%	19.96%	1.15
Sep	940	c	1 3/4	1,040	9/19/97	4	0.00	1.75	0.99944	n/a	2.46	40.7%	20.70%	0.51
Sep	945	c	1 3/8	1,783	9/19/97	4	0.00	1.38	0.99944	n/a	1.65	19.7%	22.22%	0.07
Sep	950	c	**1/2**	774	9/19/97	4	0.00	0.50	0.99944	n/a	1.07	113.0%	19.58%	0.32
Sep	955	c	**1/4**	1,896	9/19/97	4	0.00	0.25	0.99944	n/a	0.67	167.1%	19.32%	0.17
Sep	960	c	**1/8**	1,844	9/19/97	4	0.00	0.13	0.99944	n/a	0.41	224.0%	19.28%	0.08
Sep	965	c	**1/8**	717	9/19/97	4	0.00	0.13	0.99944	n/a	0.24	90.2%	21.28%	0.01
Sep	970	c	**1/8**	225	9/19/97	4	0.00	0.13	0.99944	n/a	0.13	8.0%	23.24%	0.00
Sep	975	c	**1/8**	11	9/19/97	4	0.00	0.13	0.99944	n/a	0.07	-40.7%	25.18%	0.00
Sep	985	c	**1/16**	72	9/19/97	4	0.00	0.06	0.99944	n/a	0.02	-67.7%	26.54%	0.00

(1) Exp Month	(2) Exercise Price	(3) Put/ Call	(4) Last Market Price	(5) Volume	(6) Exp Date	(7) Days to Exp	(8) Intrinsic Value	(9) Time Value	(10) Discount Factor	(11) BS Put Price	(12) BS Call Price	(13) BS Model Pricing Error	(14) Implied Volatility	(15) SSE
Oct	700	c	229 3/4	1	10/17/97	32	219.77	9.98	0.99552	n/a	222.90	-3.0%	???	46.87
Oct	885	c	53 3/4	1	10/17/97	32	34.77	18.98	0.99552	n/a	48.98	-8.9%	28.75%	22.73
Oct	895	c	44	1	10/17/97	32	24.77	19.23	0.99552	n/a	42.09	-4.3%	25.47%	3.64
Oct	900	c	40	10	10/17/97	32	19.77	20.23	0.99552	n/a	38.86	-2.8%	24.64%	1.29
Oct	905	c	40 1/4	3	10/17/97	32	14.77	25.48	0.99552	n/a	35.78	-11.1%	27.82%	19.96
Oct	910	c	37 1/2	704	10/17/97	32	9.77	27.73	0.99552	n/a	32.86	-12.4%	27.90%	21.57
Oct	910	c	50 1/2	25	10/17/97	31	35.64	14.86	0.99566	n/a	49.86	-1.3%	24.22%	0.41
Oct	920	c	27 1/2	148	10/17/97	32	0.00	27.50	0.99552	n/a	27.46	-0.1%	23.53%	0.00
Oct	925	c	24 1/4	541	10/17/97	32	0.00	24.25	0.99552	n/a	25.00	3.1%	22.81%	0.56
Oct	930	c	22 1/2	488	10/17/97	32	0.00	22.50	0.99552	n/a	22.69	0.9%	23.32%	0.04
Oct	935	c	20	11	10/17/97	32	0.00	20.00	0.99552	n/a	20.53	2.7%	23.00%	0.29
Oct	940	c	17 1/2	14	10/17/97	32	0.00	17.50	0.99552	n/a	18.53	5.9%	22.53%	1.05
Oct	945	c	17	13	10/17/97	32	0.00	17.00	0.99552	n/a	16.66	-2.0%	23.82%	0.11
Oct	950	c	14	2,329	10/17/97	32	0.00	14.00	0.99552	n/a	14.94	6.7%	22.57%	0.89
Oct	960	c	10 1/2	335	10/17/97	32	0.00	10.50	0.99552	n/a	11.91	13.4%	22.00%	1.98
Oct	970	c	7 7/8	167	10/17/97	32	0.00	7.88	0.99552	n/a	9.37	19.0%	21.75%	2.24
Oct	980	c	5 1/2	15	10/17/97	32	0.00	5.50	0.99552	n/a	7.28	32.4%	21.12%	3.18
Oct	980	c	11 3/4	1,289	10/17/97	31	0.00	11.75	0.99566	n/a	13.89	18.2%	21.35%	4.57
Oct	990	c	3 5/8	354	10/17/97	32	0.00	3.63	0.99552	n/a	5.59	54.2%	20.42%	3.87
Oct	995	c	3	53	10/17/97	32	0.00	3.00	0.99552	n/a	4.88	62.5%	20.29%	3.52
Oct	1,005	c	2 3/16	5	10/17/97	32	0.00	2.19	0.99552	n/a	3.67	67.9%	20.47%	2.21
Oct	1,010	c	2	50	10/17/97	32	0.00	2.00	0.99552	n/a	3.17	58.7%	20.91%	1.38
Oct	1,025	c	1 1/4	114	10/17/97	32	0.00	1.25	0.99552	n/a	2.01	60.8%	21.23%	0.58
Nov	880	c	66	10	11/21/97	67	39.77	26.23	0.99065	n/a	64.94	-1.6%	24.30%	1.13
Nov	900	c	56 1/4	2	11/21/97	67	19.77	36.48	0.99065	n/a	52.18	-7.2%	26.25%	16.59
Nov	920	c	44 1/2	6	11/21/97	67	0.00	44.50	0.99065	n/a	41.10	-7.6%	25.69%	11.58
Nov	925	c	39 1/2	7	11/21/97	67	0.00	39.50	0.99065	n/a	38.59	-2.3%	24.08%	0.82
Nov	950	c	28	1	11/21/97	67	0.00	28.00	0.99065	n/a	27.65	-1.3%	23.73%	0.12
Nov	980	c	16	6	11/21/97	67	0.00	16.00	0.99065	n/a	17.75	11.0%	22.24%	3.07
Nov	980	c	24	48	11/21/97	66	0.00	24.00	0.99079	n/a	26.85	11.9%	21.68%	8.11

(1) Exp Month	(2) Exercise Price	(3) Put/ Call	(4) Last Market Price	(5) Volume	(6) Exp Date	(7) Days to Exp	(8) Intrinsic Value	(9) Time Value	(10) Discount Factor	(11) BS Put Price	(12) BS Call Price	(13) BS Model Pricing Error	(14) Implied Volatility	(15) SSE
Dec	750	c	186	714	12/19/97	95	169.77	16.23	0.98677	n/a	181.06	-2.7%	32.83%	24.44
Dec	805	c	135	3	12/19/97	95	114.77	20.23	0.98677	n/a	130.90	-3.0%	27.92%	16.82
Dec	890	c	67 1/2	10	12/19/97	95	29.77	37.73	0.98677	n/a	66.89	-0.9%	23.86%	0.37
Dec	900	c	64	6	12/19/97	95	19.77	44.23	0.98677	n/a	60.88	-4.9%	25.27%	9.75
Dec	910	c	59 1/4	102	12/19/97	95	9.77	49.48	0.98677	n/a	55.21	-6.8%	25.73%	16.28
Dec	930	c	44 1/4	3,291	12/19/97	95	0.00	44.25	0.98677	n/a	44.96	1.6%	23.12%	0.50
Dec	930	c	59 1/2	1,537	12/19/97	94	15.64	43.86	0.98691	n/a	59.59	0.1%	23.45%	0.01
Dec	935	c	41 3/4	5	12/19/97	95	0.00	41.75	0.98677	n/a	42.62	2.1%	23.04%	0.75
Dec	940	c	42 1/2	264	12/19/97	95	0.00	42.50	0.98677	n/a	40.36	-5.0%	24.64%	4.56
Dec	950	c	36 1/4	14	12/19/97	95	0.00	36.25	0.98677	n/a	36.11	-0.4%	23.57%	0.02
Dec	960	c	31 1/2	2	12/19/97	95	0.00	31.50	0.98677	n/a	32.20	2.2%	23.12%	0.49
Dec	990	c	21	5	12/19/97	95	0.00	21.00	0.98677	n/a	22.37	6.5%	22.69%	1.86
Dec	995	c	20	107	12/19/97	95	0.00	20.00	0.98677	n/a	20.98	4.9%	22.91%	0.97
Dec	1,025	c	11	7	12/19/97	95	0.00	11.00	0.98677	n/a	14.07	27.9%	21.28%	9.43
Dec	1,025	c	15 1/2	30	12/19/97	94	0.00	15.50	0.98691	n/a	20.79	34.1%	20.28%	27.99

(1) Exp Month	(2) Exercise Price	(3) Put/ Call	(4) Last Market Price	(5) Volume	(6) Exp Date	(7) Days to Exp	(8) Intrinsic Value	(9) Time Value	(10) Discount Factor	(11) BS Put Price	(12) BS Call Price	(13) BS Model Pricing Error	(14) Implied Volatility	(15) SSE
Sep	800	p	1/16	510	9/19/97	4	0.00	0.06	0.99944	0.00	120.22	-100.0%	-	0.00
Sep	815	p	1/16	40	9/19/97	4	0.00	0.06	0.99944	0.00	105.23	-100.0%	-	0.00
Sep	825	p	1/8	6	9/19/97	4	0.00	0.13	0.99944	0.00	95.23	-100.0%	-	0.02
Sep	830	p	1/8	41	9/19/97	4	0.00	0.13	0.99944	0.00	90.24	-99.9%	-	0.02
Sep	840	p	3/16	110	9/19/97	4	0.00	0.19	0.99944	0.00	80.24	-99.7%	-	0.03
Sep	845	p	3/16	365	9/19/97	4	0.00	0.19	0.99944	0.00	75.25	-99.2%	-	0.03
Sep	850	p	1/4	5,243	9/19/97	4	0.00	0.25	0.99944	0.00	70.25	-98.6%	-	0.06
Sep	860	p	5/8	390	9/19/97	4	0.00	0.63	0.99944	0.02	60.27	-96.9%	-	0.37
Sep	870	p	13/16	3,365	9/19/97	4	0.00	0.81	0.99944	0.08	50.34	-89.6%	-	0.53
Sep	875	p	7/8	175	9/19/97	4	0.00	0.88	0.99944	0.16	45.42	-81.4%	-	0.51
Sep	880	p	1	889	9/19/97	4	0.00	1.00	0.99944	0.30	40.56	-69.9%	-	0.49
Sep	885	p	1 1/2	317	9/19/97	4	0.00	1.50	0.99944	0.53	35.80	-64.7%	-	0.94
Sep	890	p	1 3/4	3,208	9/19/97	4	0.00	1.75	0.99944	0.89	31.16	-49.1%	-	0.74
Sep	895	p	2	2,944	9/19/97	4	0.00	2.00	0.99944	1.44	26.71	-28.2%	-	0.32
Sep	900	p	2 11/16	3,408	9/19/97	4	0.00	2.69	0.99944	2.22	22.50	-17.3%	-	0.22
Sep	905	p	3 1/2	397	9/19/97	4	0.00	3.50	0.99944	3.31	18.59	-5.4%	-	0.04
Sep	910	p	4 3/4	1,765	9/19/97	4	0.00	4.75	0.99944	4.76	15.04	0.1%	-	0.00
Sep	915	p	6 1/4	1,583	9/19/97	4	0.00	6.25	0.99944	6.60	11.89	5.7%	-	0.13
Sep	920	p	8 1/8	2,319	9/19/97	4	0.23	7.89	0.99944	8.88	9.17	9.3%	-	0.57
Sep	925	p	10 1/4	1,278	9/19/97	4	5.23	5.02	0.99944	11.60	6.89	13.2%	-	1.82
Sep	930	p	14	2,748	9/19/97	4	10.23	3.77	0.99944	14.74	5.03	5.3%	-	0.55
Sep	935	p	**14 3/4**	280	9/19/97	4	15.23	-0.48	0.99944	18.28	3.57	23.9%	-	12.46
Sep	935	p	2	3,330	9/19/97	3	0.00	2.00	0.99958	3.66	14.69	82.8%	-	161.02
Sep	940	p	21	450	9/19/97	4	20.23	0.77	0.99944	22.17	2.46	5.6%	-	1.36
Sep	945	p	22	20	9/19/97	4	25.23	-3.23	0.99944	26.35	1.65	19.8%	-	18.89
Sep	950	p	24	56	9/19/97	4	30.23	-6.23	0.99944	30.76	1.07	28.2%	-	45.73
Sep	955	p	27 7/8	200	9/19/97	4	35.23	-7.36	0.99944	35.36	0.67	26.9%	-	56.06
Sep	960	p	35 1/2	41	9/19/97	4	40.23	-4.73	0.99944	40.10	0.41	12.9%	-	21.13
Sep	975	p	47	10	9/19/97	4	55.23	-8.23	0.99944	54.76	0.07	16.5%	-	60.18
Sep	995	p	66 1/2	10	9/19/97	4	75.23	-8.73	0.99944	74.68	0.00	12.3%	-	66.86
Sep	1,010	p	89 1/8	9	9/19/97	4	90.23	-1.11	0.99944	89.66	0.00	0.6%	-	0.29

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Oct	700	p	1/4	325	10/17/97	32	0.00	0.25	0.99552	0.00	222.90	-99.8%	-	0.06
Oct	750	p	1	428	10/17/97	32	0.00	1.00	0.99552	0.02	173.15	-97.8%	-	0.96
Oct	800	p	2 3/8	107	10/17/97	32	0.00	2.38	0.99552	0.42	123.77	-82.4%	-	3.83
Oct	825	p	3 3/4	23	10/17/97	32	0.00	3.75	0.99552	1.32	99.78	-64.9%	-	5.92
Oct	830	p	4 1/8	1	10/17/97	32	0.00	4.13	0.99552	1.62	95.10	-60.7%	-	6.28
Oct	840	p	4 5/8	123	10/17/97	32	0.00	4.63	0.99552	2.40	85.93	-48.2%	-	4.96
Oct	850	p	6 3/8	333	10/17/97	32	0.00	6.38	0.99552	3.46	77.03	-45.8%	-	8.53
Oct	860	p	6 3/4	110	10/17/97	32	0.00	6.75	0.99552	4.85	68.47	-28.1%	-	3.59
Oct	870	p	8 3/4	78	10/17/97	32	0.00	8.75	0.99552	6.66	60.32	-23.9%	-	4.38
Oct	880	p	11 3/8	674	10/17/97	32	0.00	11.38	0.99552	8.93	52.63	-21.5%	-	6.00
Oct	885	p	12	45	10/17/97	32	0.00	12.00	0.99552	10.25	48.98	-14.6%	-	3.06
Oct	890	p	13 1/5	58	10/17/97	32	0.00	13.20	0.99552	11.71	45.47	-11.3%	-	2.21
Oct	895	p	14 7/8	501	10/17/97	32	0.00	14.88	0.99552	13.32	42.09	-10.5%	-	2.43
Oct	900	p	16 1/2	583	10/17/97	32	0.00	16.50	0.99552	15.06	38.86	-8.7%	-	2.06
Oct	905	p	16 1/2	18	10/17/97	32	0.00	16.50	0.99552	16.96	35.78	2.8%	-	0.21
Oct	910	p	19	89	10/17/97	32	0.00	19.00	0.99552	19.01	32.86	0.1%	-	0.00
Oct	915	p	21 7/8	30	10/17/97	32	0.00	21.88	0.99552	21.22	30.08	-3.0%	-	0.43
Oct	920	p	23 1/2	177	10/17/97	32	0.23	23.27	0.99552	23.58	27.46	0.3%	-	0.01
Oct	925	p	26 1/2	119	10/17/97	32	5.23	21.27	0.99552	26.09	25.00	-1.5%	-	0.17
Oct	930	p	28	745	10/17/97	32	10.23	17.77	0.99552	28.76	22.69	2.7%	-	0.58
Oct	950	p	39 1/4	141	10/17/97	32	30.23	9.02	0.99552	40.92	14.94	4.3%	-	2.80
Oct	960	p	43 1/4	40	10/17/97	32	40.23	3.02	0.99552	47.84	11.91	10.6%	-	21.07
Oct	990	p	66 1/4	16	10/17/97	32	70.23	-3.98	0.99552	71.39	5.59	7.8%	-	26.42
Oct	995	p	67	10	10/17/97	32	75.23	-8.23	0.99552	75.65	4.88	12.9%	-	74.86
Oct	1,025	p	101 1/2	9	10/17/97	32	105.23	-3.73	0.99552	102.65	2.01	1.1%	-	1.33

(1) Exp Month	(2) Exercise Price	(3) Put/ Call	(4) Last Market Price	(5) Volume	(6) Exp Date	(7) Days to Exp	(8) Intrinsic Value	(9) Time Value	(10) Discount Factor	(11) BS Put Price	(12) BS Call Price	(13) BS Model Pricing Error	(14) Implied Volatility	(15) SSE
Nov	700	p	1 1/8	45	11/21/97	67	0.00	1.13	0.99065	0.06	226.37	-94.7%	-	1.13
Nov	750	p	2 3/8	10	11/21/97	67	0.00	2.38	0.99065	0.51	177.29	-78.6%	-	3.48
Nov	800	p	5 5/8	213	11/21/97	67	0.00	5.63	0.99065	2.64	129.89	-53.1%	-	8.91
Nov	810	p	7	10	11/21/97	67	0.00	7.00	0.99065	3.49	120.83	-50.1%	-	12.31
Nov	820	p	7 1/4	1	11/21/97	67	0.00	7.25	0.99065	4.55	111.99	-37.2%	-	7.29
Nov	830	P	8 3/4	303	11/21/97	67	0.00	8.75	0.99065	5.85	103.38	-33.2%	-	8.43
Nov	840	p	10 3/8	2	11/21/97	67	0.00	10.38	0.99065	7.41	95.03	-28.6%	-	8.78
Nov	850	p	12 3/4	7	11/21/97	67	0.00	12.75	0.99065	9.28	86.99	-27.2%	-	12.07
Nov	880	p	17 1/2	3	11/21/97	67	0.00	17.50	0.99065	16.94	64.94	-3.2%	-	0.31
Nov	900	p	25	26	11/21/97	67	0.00	25.00	0.99065	23.99	52.18	-4.0%	-	1.01
Nov	920	p	29 5/8	6	11/21/97	67	0.23	29.40	0.99065	32.73	41.10	10.5%	-	9.62
Nov	925	p	32 1/2	2	11/21/97	67	5.23	27.27	0.99065	35.18	38.59	8.2%	-	7.16
Nov	930	p	37	4	11/21/97	67	10.23	26.77	0.99065	37.73	36.20	2.0%	-	0.54
Nov	940	p	42	10	11/21/97	67	20.23	21.77	0.99065	43.16	31.72	2.8%	-	1.34

(1) Exp Month	(2) Exercise Price	(3) Put/ Call	(4) Last Market Price	(5) Volume	(6) Exp Date	(7) Days to Exp	(8) Intrinsic Value	(9) Time Value	(10) Discount Factor	(11) BS Put Price	(12) BS Call Price	(13) BS Model Pricing Error	(14) Implied Volatility	(15) SSE
Dec	550	p	**3/16**	75	12/19/97	95	0.00	0.19	0.98677	0.00	377.05	-99.9%	-	0.04
Dec	575	p	5/16	55	12/19/97	95	0.00	0.31	0.98677	0.00	352.38	-99.8%	-	0.10
Dec	650	p	15/16	20	12/19/97	95	0.00	0.94	0.98677	0.03	278.40	-96.3%	-	0.82
Dec	675	p	1 3/8	50	12/19/97	95	0.00	1.38	0.98677	0.10	253.80	-92.6%	-	1.62
Dec	700	p	2 3/8	88	12/19/97	95	0.00	2.38	0.98677	0.27	229.30	-88.7%	-	4.44
Dec	725	p	3 1/4	1	12/19/97	95	0.00	3.25	0.98677	0.64	205.00	-80.4%	-	6.84
Dec	750	p	4 7/8	191	12/19/97	95	0.00	4.88	0.98677	1.36	181.06	-72.0%	-	12.32
Dec	775	p	6 1/8	46	12/19/97	95	0.00	6.13	0.98677	2.69	157.71	-56.1%	-	11.82
Dec	780	p	6 1/4	10	12/19/97	95	0.00	6.25	0.98677	3.05	153.14	-51.2%	-	10.25
Dec	800	p	8 1/4	31	12/19/97	95	0.00	8.25	0.98677	4.90	135.25	-40.6%	-	11.22
Dec	820	p	11 5/8	2	12/19/97	95	0.00	11.63	0.98677	7.54	118.16	-35.2%	-	16.71
Dec	825	p	12	7	12/19/97	95	0.00	12.00	0.98677	8.34	114.02	-30.5%	-	13.39
Dec	830	p	13 1/2	3	12/19/97	95	0.00	13.50	0.98677	9.21	109.96	-31.8%	-	18.44
Dec	840	p	14 1/2	1	12/19/97	95	0.00	14.50	0.98677	11.14	102.02	-23.2%	-	11.31
Dec	850	p	17	195	12/19/97	95	0.00	17.00	0.98677	13.35	94.36	-21.5%	-	13.33
Dec	875	p	23 3/4	24	12/19/97	95	0.00	23.75	0.98677	20.23	76.57	-14.8%	-	12.40
Dec	890	p	26	12	12/19/97	95	0.00	26.00	0.98677	25.35	66.89	-2.5%	-	0.42
Dec	900	P	31 1/2	7,028	12/19/97	95	0.00	31.50	0.98677	29.20	60.88	-7.3%	-	5.29
Dec	910	p	32 3/4	1	12/19/97	95	0.00	32.75	0.98677	33.41	55.21	2.0%	-	0.43
Dec	920	p	37 3/4	11	12/19/97	95	0.23	37.52	0.98677	37.97	49.91	0.6%	-	0.05
Dec	925	p	39 1/2	46	12/19/97	95	5.23	34.27	0.98677	40.38	47.39	2.2%	-	0.78
Dec	930	p	42 5/8	2,489	12/19/97	95	10.23	32.40	0.98677	42.89	44.96	0.6%	-	0.07
Dec	935	p	43 1/2	4	12/19/97	95	15.23	28.27	0.98677	45.48	42.62	4.5%	-	3.92
Dec	940	p	481/4	268	12/19/97	95	20.23	28.02	0.98677	48.16	40.36	-0.2%	-	0.01
Dec	950	p	511/4	3	12/19/97	95	30.23	21.02	0.98677	53.78	36.11	4.9%	-	6.38
Dec	960	p	55	1	12/19/97	95	40.23	14.77	0.98677	59.73	32.20	8.6%	-	22.37
Dec	965	p	573/4	1,020	12/19/97	95	45.23	12.52	0.98677	62.83	30.37	8.8%	-	25.81
Dec	970	p	62	28	12/19/97	95	50.23	11.77	0.98677	66.01	28.61	6.5%	-	16.09
Dec	975	p	661/2	11	12/19/97	95	55.23	11.27	0.98677	69.27	26.94	4.2%	-	7.67
Dec	1,050	p	127	10	12/19/97	95	130.23	-3.23	0.98677	126.20	9.86	-0.6%	-	0.64